Secondary mathematics teachers’ instrumental integration in technology-rich geometry classrooms

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**Abstract**

Instrumental genesis, a psychological construct that describes the process of how an artifact becomes an instrument, illuminates the ways technological tools support mathematics learning. Teachers have vital roles in designing suitable tasks, assisting students in making connections between their work with the artifact and the mathematics they are learning, and making careful teaching moves in organizing instrumental genesis. The current study examines secondary mathematics teachers’ instrumental integration when they utilize student-created dynamic geometry sketches in technology-rich classrooms. The results demonstrate that some teachers view moments when students experienced technological or mathematical confusion as an opportunity for mathematical learning while other teachers maintained a focus on technology. Implications for future research are provided.

**1. Introduction**

Students’ learning of mathematics is influenced by the tasks they are given (Stein, Grover, & Henningsen, 1996). It is not uncommon for such tasks to involve the use of tools (Bartolini Bussi & Mariotti, 2008; Trouche, 2004). Tools include technology such as graphing calculators and dynamic geometry programs. Tools can also be non-technological like manipulatives, algebra tiles, geoboards, and compasses. Some researchers call technological tools (e.g., dynamic geometry software) artifacts (Bartolini Bussi & Mariotti, 2008).

When using dynamic geometric software (DGS), teachers often create a sketch that contains important geometric objects for students to investigate. Teachers may also choose to begin with a blank sketch and allow students to create their own geometric objects (Dove & Hollebrands, 2014; Erfjord, 2011; Ruthven, Hennessy, & Deane, 2008). We refer to this type of sketch as student-created and the former type of sketch as teacher-created or pre-constructed. In a student-created sketch, students start with a blank DGS sketch and use their knowledge of the technology to create geometric objects (Dove & Hollebrands, 2014; Erfjord, 2011).

The construction process of a mathematical object in DGS may challenge students because they need to use mathematical properties skillfully, and DGS “does not hide the complexity of the situation as the paper and pencil environment does” (Laborde, 1998, p.193). Teachers have dual roles when they use a student-created sketch. For example, teachers may focus on technology and give students little opportunity for mathematical learning. On the other hand, they may focus mostly on the mathematics and not provide any technology instructions at all (Assude, 2007; Erfjord, 2011). Therefore, it is important to understand how teachers utilize student-created sketches when they introduce a new tool to students. In particular, it is important to understand how teachers connect mathematical concepts and technology when students have access to technological tools in the mathematics classroom.

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http://dx.doi.org/10.1016/j.jmathb.2017.10.003
Received 9 January 2017; Received in revised form 25 October 2017; Accepted 30 October 2017
0732-3123/ © 2017 Published by Elsevier Inc.

Please cite this article as: Hollebrands, K., Journal of Mathematical Behavior (2017),
http://dx.doi.org/10.1016/j.jmathb.2017.10.003
One of the common frameworks used to describe how students use tools is instrumental genesis (Vérillon & Rabardel, 1995). Instrumental genesis examines the process by which an artifact becomes an instrument (Vérillon & Rabardel, 1995). An artifact refers to the physical or symbolic object, algorithms etc. while the instrument refers to the ways in which the tool has been incorporated into the thinking of the user. During this process, students’ utilization schemes and techniques play a central role. Utilization schemes are mental acts involved with using an artifact, and only visible by the techniques students use when they interact with the artifact.

Instrumental genesis focuses on an individual and his or her relationships to an artifact. It does not focus on the teacher and classroom and the important social interactions that take place around students’ uses of tools (Bartolini Bussi & Mariotti, 2008). Researchers have built upon the theory of instrumental genesis to consider the important role of the teacher (e.g., Assude, Grugeon, Laborde, & Soury-Lavergne, 2006; Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Trouche, 2004). In particular, instrumental integration highlights how a teacher utilizes instrumental genesis (Assude et al., 2006; Assude, 2007; Goos et al., 2010). When a student-created sketch is implemented, it may be easy to determine if the teachers’ main focus is on the technology if technological directions are explicitly provided or if students’ knowledge of the technology appears to be the goal a teacher wants to address. However, if the student-created sketch requires the use of mathematical and technological knowledge to accomplish the task, teachers’ instrumental integration may be less clear. The purpose of the current research study is to consider secondary mathematics teachers’ uses of instrumental integration when implementing The Geometer’s Sketchpad (GSP) in technology-rich classrooms. Our research question is: “How can we characterize mathematics teachers’ instrumental integration when they utilize student-created sketches in technology-rich classrooms?”

1.1. Theoretical background

Researchers proposed instrumental integration as a theory for characterizing how teachers organize instrumental genesis in mathematics classrooms (Assude et al., 2006; Assude, 2007; Goos et al., 2010). It is important to know the ways mathematics teachers support students’ instrumental genesis because research studies indicate that tools influenced by instrumental genesis serve as means to students’ accomplishment of a task (e.g., McCulloch, 2011; Tabach, Hershkowitz, & Arcavi, 2008). As McCulloch (2011) emphasized, if a technological tool is not “developed as an instrument with respect to the concepts at hand, it is useless and can even have adverse effects on one’s affective pathway” (p.177). Teachers make decisions about how to use an artifact based on their perceptions of students’ familiarity with the tool. Assude et al. (2006) described four ways in which teachers support instrumental integration. During instrumental initiation, teachers introduce students to the artifact, particularly if they have little experience using it.

Instrumental exploration is described as the embedding of an artifact in a mathematical activity to support students’ learning of mathematics while they also learn to use the artifact. Tasks created for instrumental exploration may include directions about how to use the artifact and “the relation between IK [instrumental knowledge] and MK [mathematical knowledge] can vary from the minimum to the maximum according to the mathematical task” (Assude, 2007, p.1342). Instrumental initiation and instrumental exploration are “lower” modes of instrumental integration and students are typically just beginning to use the artifact (Assude et al., 2006; Assude, 2007).

With instrumental reinforcement, a mathematical task is presented that requires students to use their mathematical and/or instrumental knowledge to address a challenge. Assude (2007) characterizes instrumental reinforcement when “pupils are already used to the software but they are confronted with instrumental difficulties while dealing with a mathematical task” (p.1342). In instrumental reinforcement, students’ difficulties may prompt teachers to address instrumental information. Instrumental symbiosis focuses on mathematical and instrumental knowledge where there is a strong relationship between them. Assude (2007) characterizes instrumental symbiosis when “pupils have already used the software and they are confronted with mathematical tasks which allow them to improve both their IK and MK because they are connected” (p.1342). Instrumental reinforcement and instrumental symbiosis are “upper” modes of instrumental integration and students typically have experience using the artifact (Assude, 2007).

Assude et al. (2006) identify the focus of the task (tool or mathematics), techniques to accomplish the task (instrumental abilities or mathematical knowledge) and interrelations between the techniques. For example, the focus of the task can be technology (tool) and/or mathematics (Assude, 2007). The descriptions of instrumental integration are correlated with didactic tetrahedron (Fig. 1a) where relationships among mathematics, students, teachers, and technology are characterized (Olive et al., 2010). Hollebrands and
Lee (2016) used the didactic tetrahedron to describe how teachers focused students’ attention while they were implementing a technology-based task. If the teacher focuses students on the technology, then we would locate this interaction on the “student – teacher – technology” face (Fig. 1a). If the teacher focuses students on mathematics, then we would describe that interaction as belonging to the “student – mathematics – teacher” face (Fig. 1a). Focus on technology involves teacher actions such as giving information about a teacher-created sketch beforehand, and creating and/or measuring geometric objects. Focus on mathematics involves little use of technology such as constructing a mathematical argument and recalling a definition/property (Hollebrands & Lee, 2016).

Often when teachers are using technology they pose questions that focus students’ attention on mathematics and technology. The faces of the didactic tetrahedron may not fully uncover the relationships between student, teacher, technology and mathematics, in particular if the task requires using mathematical and technological knowledge. For example, the teacher can ask a mathematical question assuming students will use technology to seek an answer or the teacher can ask a technological question expecting students to notice a mathematical idea (Hollebrands & Lee, 2016). Hollebrands and Lee (2016) called the former focus on mathematics with the use of technology and the latter focus on technology to notice mathematics. If the didactical tetrahedron (Fig. 1a) is unfolded from the technology vertex as shown in Fig. 1b, focus on mathematics with the use of technology follows “teacher – mathematics – technology – student” (see Fig. 2a) and is identified when “the teacher makes a statement or asks the student a mathematical question with the assumption that the student will do something with the technology to respond” (p.152). For example, teachers may exploit mathematical knowledge with the use of technology such as focusing on variance/invariance and making a connection between mathematics and technological representations.

On the other hand, teachers may help students notice mathematical relationships by focusing students’ attention on certain actions, measures, or objects in technological tools. Focus on technology to notice mathematics follows “teacher – technology – mathematics – student” as shown in Fig. 2b and is identified when “the teacher makes a statement or poses a question related to a technological action that is related to a mathematical idea” (p.152). In this respect, the unfolded tetrahedron gives a stronger lens in identifying the relationship between student, teacher, mathematics and technology, and enriches characterizing teachers’ use of instrumental integration.

1.2. Student-created technological tasks

Utilization schemes and techniques are influenced by affordances and constraints of the artifact (instrumentalization) and methods that students utilize (instrumentalization) during the process in which an artifact becomes an instrument. To illustrate instrumentalization, a student can construct a rhombus in DGS using a circle and parallel lines. Another student can follow a different approach constructing a generic isosceles triangle and reflect the congruent sides about the third side of the triangle. When a point of either construction is dragged, the way it behaves provides feedback to the student that is related to the properties of a rhombus (instrumentation) (Goos et al., 2010). Students’ understandings of the artifact and a rhombus influence their approach and the behavior of the constructed rhombus influences how they think about the properties of this object.

It is not uncommon for students to have difficulty accomplishing construction tasks. Healy, Hoelzl, Hoyles, and Noss (1994) found that messing up a construction while students grappled with DGS to overcome difficulties constructing turned out to be a powerful idea. Some students managed to make robust constructions using mathematical properties skillfully whereas others produced soft constructions using mathematical properties less skillfully or poorly (Healy, 2000). In addition, some students made overconstrained constructions resulting in special cases of a construction such as constructing a square when asked to construct a rhombus (Finzer & Bennett, 1995; Scher, 2005). On the other hand, students can make underconstrained constructions where the construction is only valid under some circumstances (Finzer & Bennett, 1995; Scher, 2005). For example, the student can construct a parallelogram and adjust its sides to be congruent to make a rhombus, but it will not always be a rhombus when a vertex is dragged.

To support students in making constructions, the teacher can provide step-by-step explanations or directions on how to construct objects in a written or oral form with a focus on instrumentalization in a student-created sketch (Assude, 2007; Cayton, Hollebrands, Okumuş, & Boehm, 2017; Erfjord, 2011). Buteau and Muller (2014) found that a tutor spent his first four lab sessions of a college programming course and gave step-by-step directions. These instructions in the lower modes of instrumental integration followed by
upper-mode instrumental integration programming language tasks. However, using programming language as an instrument was found to be a slow process for some college students (Buteau, Muller, Marshall, Sacristán, & Mgombelo, 2016).

Teachers can allow students to share his/her construction steps to the whole class. Trouche (2004) described a student in this role as a Sherpa. Alternatively, the teacher can ask the Sherpa student to follow his or her directions in a classroom environment where all students observe the student’s construction (Drijvers et al., 2010). Drawing upon instrumental genesis, Drijvers et al. (2010) describe instrumental orchestration as “the intentional and systematic organization of the various artifacts available in a computerized learning environment by the teacher for a given mathematical situation, in order to guide students’ instrumental geneses” (p.112). According to Trouche (2004), instrumental orchestration is consisted of didactic configurations which involve arrangements and configurations of the teacher in the learning environment (e.g. having students work in dyads), and exploration modes in which the teacher decides to enact her intentions in accordance with his/her didactic configurations.

Drijvers et al. (2010) add didactical performance to instrumental orchestration in which “ad hoc decisions taken while teaching on how to actually perform in the chosen didactic configuration and exploitation mode” (p.215). They found the six types of instrumental orchestration during three mathematics teachers’ enactment of dynamic java applets: Technical-demonstration, Explain-the-screen, Link-screen-board, Discuss-the-screen, Spot-and-show (the student’s reasoning or work in technology is shared in the mathematics culture), and Sherpa-at-work. The researchers characterized the first three orchestrations as teacher-centered orchestrations, and last three orchestrations as student-centered orchestrations. However, the Sherpa-at-work orchestration as Drijvers et al. (2010) characterize may not allow for developing any mathematical knowledge in the lower modes of the instrumental integration. In other words, this student-centered orchestration may guide students to develop their instrumental abilities if the teacher asks the Sherpa student to “carry out specific actions in the technological environment” (p.220).

Assude (2007) observed that students who followed the teacher’s technological directions had little understanding about the global purpose of the activity. Such teacher-dominated actions limit students’ employing their own methods to overcome the instrumental difficulty (Assude, 2007; Erfjord, 2011). Talmon and Yerushalmy (2004) asked students to follow written directions of two similar constructions. However, the order of constructions in mathematical objects differed. Students were encouraged to predict the outcomes of dragging points in the constructions prior to the points were dragged. The research study demonstrated that students had difficulty predicting the dynamic behaviors of constructions (e.g., the order of constructions) in student-created dynamic sketches and the researchers raised questions about different DGS interfaces.

Previous research studies indicated that students made constructions with a limitation or did freehand constructions (Mariotti, 2013; Ruthven et al., 2008). In these research studies, teachers used these constructions as an opportunity to draw an attention to mathematical learning and generated a whole-class discussion (Mariotti, 2013; Ruthven et al., 2008). Ruthven et al. (2008) described these types of actions as capitalizing. However, there is little research focused on teachers’ enactment of student-created tasks in which the tool is new to students. Because student-created sketches require students to apply their knowledge of mathematics and technology, teachers may pursue different types of instrumental integration to “organize the conditions for instrumental genesis” (Goos et al., 2010, p.313).

2. Methods

A case study was designed to examine mathematics teachers’ instrumental integration when they utilized student-created geometry sketches (Merriam, 2009). According to Merriam (2009) “a qualitative case study is an intensive, holistic description and analysis of a bounded phenomenon such as a program, an institution, a person, a process, or a social unit” (p. X). Case studies are bounded by time, place, people or by other phenomena. Merriam (2009) stressed:

One technique for assessing the boundedness of the topic is to ask how finite the data collection would be…If there is no end, actually or theoretically, to the number of people who could be interviewed or to observations that could be conducted, then the phenomenon is not bounded enough to qualify as a case” (p.41).

The data for the current study were collected from a bounded system in which mathematics teachers participated in a two-year professional development (PD) project. The PD focused on teaching and learning mathematics with The Geometer’s Sketchpad (GSP)
with an emphasis on topics from algebra and geometry in one-to-one laptop classrooms in which one laptop was given to each student. However, there were times students in small groups shared a laptop. Mr. Sanders, Mr. Jones, Mr. Taylor, Mrs. Dabney, Mrs. Snyder represent five teachers in the PD (the cases). The cases were selected for understanding the phenomenon of teachers’ instrumental integration during their enactment of student-created sketches. In other words, teachers’ enactment of student-created sketches characterized the current study.

Teachers had little experience in using GSP before the professional development. GSP is a dynamic geometry program that allows learners to explore geometric relationships between objects as a result of dragging its primitive elements. The primitive tools such as points and circles are available to use on the toolbar that are on the left side. There are primitive objects available from the dropdown menu in the toolbar (e.g., the Ray tool). Construction, measurement tools etc. are available in the menu bar that is on the upper side. However, GSP does not allow students to use them unless the necessary input is provided. For example, if students want to construct a parallel line to a segment, the segment and a point that the parallel line will pass through should be selected (Fig. 2a). Otherwise, the Parallel Line tool will be faded and unavailable to use in the Construct dropdown menu (Fig. 3b).

The professional development activities included creating and critiquing existing tasks. Teachers created their own tasks and modified existing tasks in a collaborative learning environment. These activities were important because they were an opportunity for teachers to improve their knowledge of the technology. Mrs. Dabney, Mrs. Snyder, and Mr. Jones were experienced teachers (more than 6 years of experience). Mr. Taylor and Mr. Sanders had fewer years of teaching experience.

2.1. Data collection and analysis

Teachers were observed during semesters they taught high school geometry. All teachers taught in schools that used block scheduling. Thus, the geometry classes met 90 min each day for one semester. Mrs. Dabney, Mr. Sanders and Mr. Taylor were observed two semesters. Mr. Jones and Mrs. Snyder were observed three semesters. Mrs. Snyder utilized five student-created sketches and Mr. Sanders utilized four student-created sketches. Two student-created sketches were collected from each of the remaining teacher.

A video camera was used to capture students’ work and teachers’ uses of technology. We identified teacher- and student-created tasks in which students used GSP. For the current research study, we analyzed the tasks in which teachers enacted student-created sketches. Also, in some lessons, teachers used the teacher computer with GSP and a projector for the purpose of demonstrating an idea. We excluded these lessons from the analyses. We adopted a deductive analysis “where the data are analyzed according to an existing framework” (Patton, 2002, p. 453). Teachers’ organization of instrumental genesis with regard to instrumental integration was characterized (Assude et al., 2006; Assude, 2007; Goos et al., 2010). Assude et al. (2006) list the following criteria to assess teachers’ instrumental integration:

- focus of the tasks given to pupils: is the focus on the tool or on mathematics?
- techniques for solving these tasks: do they come mainly within mathematics or technology or within both, or in other words does the task require mainly instrumental abilities (IA) or mainly mathematical knowledge (MK), or both are equally important?
- extent of the intertwining of instrumental abilities and mathematical knowledge involved in the task (IA/MK) (p. 319).

Identification of the focus of the task as technology (tool) or mathematics may not capture the ways teachers organize the conditions for instrumental genesis. For example, teachers may refer to a technological representation assuming students will notice a mathematical idea underlying the representation. Using the codes of unfolded didactical tetrahedron gave a new lens to the researchers to depict the modes of instrumental integration. In order to determine the teachers’ instrumental integration, we used a segmenting method by dividing the data into chunks (Merriam, 2009). According to Merriam (2009), “a unit of data is any meaningful (or potentially meaningful) segment of data; at the beginning of a study the researcher is uncertain about what will ultimately be meaningful” (p.176).

Video recordings were analyzed chronologically. Segments (modes of instrumental integration) that involved the use of
technology to accomplish the tasks were coded by individual researchers. We coded the data with regard to unfolded didactical tetrahedron to identify the segments of data: focus on technology, focus on mathematics, focus on mathematics with the use of technology and focus on technology to notice mathematics (Hollebrands & Lee, 2016). However, we report few cases in which teachers focused on mathematics. We met to compare codes and when there were differences, the researchers discussed the segment of data and reached a consensus on the codes and/or modes of instrumental integration. We discussed the relationships between mathematical and instrumental knowledge within the segments of data. Some of the questions around how teachers organized the conditions for instrumental genesis we considered were: Did students know how to use the tool(s) in GSP prior to the task? Was the mathematical knowledge new for students? Was the knowledge of how to use a certain tool in GSP (instrumental knowledge) a necessity for students to accomplish the task? Did the task require using mathematical knowledge to identify what tool in GSP should be used? Such questions helped the researchers identify teachers’ modes of instrumental integration.

We characterized teachers’ uses of instrumental integration in the upper and lower modes to identify how they organized the conditions for instrumental genesis when they enacted student-created sketches. A segment of data was identified by one of the lower modes of instrumental integration if the focus of the teachers was the instrumental abilities. For example, when teachers focused on technology and introduced a new tool in GSP in a way that was disconnected to mathematical knowledge, it was characterized instrumental initiation.

When teachers focused on mathematical knowledge using technology, upper modes of instrumental integration were characterized. Nonetheless, in instrumental exploration (a lower mode) and instrumental reinforcement (an upper mode), students may encounter instrumental difficulty. If mathematical knowledge was not new for students and if teachers required students to find the tool(s) in GSP to accomplish the task, the mode was identified as instrumental exploration. In this mode, developing mathematical knowledge was not primary focus of mathematics teachers. On the other hand, if teachers posed a question in a way that students were required using and developing a piece of mathematical knowledge while they investigate a tool(s) in GSP, the mode was identified as instrumental reinforcement.

There were segments of data in which teachers intended to develop students’ mathematical knowledge by building on students’ instrumental abilities and existing mathematical knowledge. Such segments of data were identified as instrumental symbiosis.

Instrumental abilities are required to accomplish a task when teachers focus on technology. On the other hand, to varying degrees, instrumental abilities and mathematical knowledge are utilized to solve a task when teachers focus on mathematics with the use of technology or technology to notice mathematics. If teachers linked mathematics with technological representations or created a construction using mathematical properties, we coded the data as focus on mathematics with the use of technology. We coded a segment of data as focus of technology to notice mathematics when a teacher used or introduced a tool in DGS to help students notice a mathematical idea (e.g., meeting conditions of a construction) that might include specifying the condition or performing a specific action. Identifying the focus of tasks, techniques, and interplay between instrumental and mathematical knowledge (Assude et al., 2006), we labeled the segments of the data that contained codes with one of the modes of instrumental integration.

3. Results

This section outlines the findings from the modes of instrumental integration mathematics teachers utilized: instrumental initiation, instrumental exploration, instrumental reinforcement and instrumental symbiosis.

3.1. Instrumental initiation

During instrumental initiation, teachers assisted students in instrumental knowledge and focused on technology. There were minimal relationships between instrumental and mathematical knowledge. We identified two methods of instrumental initiation: providing step-by-step technology directions and giving a responsibility to a Sherpa student. In the first instrumental initiation activity,
teachers provided written and/or oral technology directions to students. For example, Mrs. Dabney provided written directions and assisted students in following her directions. She assisted those who had difficulty following the written directions. We identified instances in which students failed to give the right input to GSP in order to use the construction tools (e.g., the Perpendicular Line tool). In those cases, teachers provided technology directions.

In most cases the other teachers generated a whole-class discussion and demonstrated tools in the menu bar/toolbar. They utilized teacher-centered approaches to develop students’ instrumental knowledge with little emphasis on mathematical knowledge. For example, Mr. Taylor gave an alternative way of constructing a right triangle using the Circle tool (his first method of a right triangle construction is given in the next section). He directed the students to create a circle and a line that passed through the center of the circle (Fig. 4a). Then, they marked a random point on the circle and connected the three points using the Segment tool. They marked the angle (Fig. 4b) and Mr. Taylor dragged the vertices of the triangle to demonstrate the right angle did not change (Fig. 4c). One of the students asked: “why does it do that?” Mr. Taylor stated: “we’re gonna cover that in circles. So, later when we cover circles, you’ll know exactly why that works.” As this vignette indicates, improving techniques required to solve the task using instrumental abilities was the focus.

In the second instrumental initiation activity, teachers invited a (Sherpa) student to use the computer that was hooked up to the projector. Teachers asked the Sherpa student to follow their directions, so other students improve their instrumental knowledge. Teachers utilized student- and/or teacher-centered approaches. For example, Mrs. Snyder provided the definition of a parallelogram along with the written directions about constructing a parallelogram as shown in Fig. 5 and asked the Sherpa student to perform the written steps. On the other hand, Mr. Taylor allowed the Sherpa student to do the construction of the Pythagorean theorem dissection. The Sherpa student did the construction on his computer following the written directions and was comfortable with demonstrating his construction to his classmates on the board. When teachers utilized instrumental initiation, they improved students’ instrumental knowledge providing step-by-step technology directions. Also, they invited a Sherpa student who followed the oral or written directions of teachers. Teachers focused on technology and the relationship between mathematical and instrumental knowledge was minimal.

### 3.2. Instrumental exploration

In this instrumental integration, teachers primarily aimed at improving instrumental knowledge. However, they associated some specific actions/instrumental knowledge with mathematical knowledge. We identified two methods of instrumental exploration: performing specific actions and exploring the tools in the toolbar/menu bar. In the first instrumental exploration activity, teachers showcased new tools in GSP with a focus on improving instrumental knowledge. There were times in which learning new tools resulted in noticing mathematical relationships/facts that was coded as focusing on technology to notice mathematics. For example, Mr. Taylor constructed a right triangle by drawing a line segment and constructing a perpendicular line to the one of the endpoints of the line segment (Fig. 6a). Until then, he provided step-by-step technology directions to students to create a right triangle and focused on technology. He made a transition to instrumental exploration when demonstrated the right triangle preserved its critical attribute when its vertices were dragged. He introduced the Marker tool to the students for the first time and pointed at the right angle. In the GSP, a common way of measuring an angle is made with the Angle tool by clicking three points. However, Mr. Taylor used the visual representation the Marker tool gave as an opportunity for measuring the angle and said: “Look it makes it easy for me (Fig. 6b). I don’t even have to measure it. It draws the box in.” Techniques Mr. Taylor utilized to solve the task was mainly instrumental. However, his construction allowed for making a connection with the invariance of the right angle. He focused on specific actions and added:

Again, the Marker tool. I clicked on the [vertex] point and I dragged it to the middle. I let it go and it drew the box for the angle. Now look no matter where I move these points, it’ll always remain a right triangle. Still right (Fig. 6c), still right. Still right, still right (he drags the vertices of the right triangle). No matter what I do to it, it is a right triangle.

In the second instrumental exploration activity, some teachers asked students to create mathematical objects without specifying what tool in GSP was required to use in their written or oral directions. In other words, students needed to think mathematically to identify what tool they could use in order to accomplish the given tasks. Then, teachers focused on mathematics with the use of
technology. For example, Mrs. Dabney asked students to create acute angles and find their measures after they defined an acute angle (Fig. 7). Mrs. Dabney had some mathematical discussion with students on what tool they were supposed to use as follows:

Mrs. Dabney: Remember, what is an angle made up of?

Student 1: I know a vertex.

Mrs. Dabney: Uh-huh. What else?

Student 1: Rays?

Mrs. Dabney: Uh-huh. Two rays. So, you could exactly do that. A vertex (puts a point). And then, hold this down (presses on the Segment tool and scrolls over the Ray tool). You see that’s a ray. And it will shoot of the page but that’s fine. And then we stop here. And then we go here.

Student 2: The two dots stop here, right?

Mrs. Dabney: Two, right. Well, you’re right. It continues to go. That’s the endpoint. Like if I wanted to name this. This is K, L, M. Well, you can change (rename) it to whatever. But you know, a ray it means it continues to go.

As it can be seen from the above excerpt, students were expected to construct a point and two rays that passed through the point. However, students did not know how to access the Ray tool. Mrs. Dabney took the responsibility of demonstrating the Ray tool and drawing an acute angle. Students were asked to recall what an angle made up of, so they could identify what tool to use, which demonstrated focusing on mathematics with the use of technology. Also, Mrs. Dabney constructed an argument about the representation of a ray in GSP. For example, while showcasing the Ray tool to a student, she posited: “if you come over here, and you choose the Ray tool. Now, you see it shoots of the page, that’s fine cause it’s a ray.” Mrs. Dabney might have noted this because a representation of ray in GSP was different from the conventional representation of a ray. Techniques utilized to solve the task were mainly instrumental as students recited or recalled the definition of a ray. On the other hand, she provided step-by-step technology directions as described in instrumental initiation to some students while she monitored the progress and noticed that some students could not accomplish the task.

When teachers utilized instrumental exploration, they improved students’ instrumental knowledge during a task that required using mathematical knowledge. New tools in GSP were introduced to students as they created sketches to accomplish the tasks. In focusing on technology to notice mathematics, teachers linked technological representations and mathematics. In focusing on the technology with the use of mathematics, teachers helped students reason about mathematical objects to identify what tool in GSP was required to use. Mathematical knowledge was not new for students as they recited or recalled a mathematical property.

3.3. Instrumental reinforcement

By definition, students are confronted with an instrumental difficulty in instrumental reinforcement in which teachers aim to improve students’ mathematical knowledge by closely relating it to instrumental knowledge. We observed one instance of instrumental reinforcement in which Mr. Jones focused on mathematics with the use of technology. Students were asked to construct a rhombus and square. They were familiar with using Perpendicular Line and Parallel Line tools in GSP. However, they were not introduced to the Circle tool that would allow them to create congruent sides. Mr. Jones initiated the task by saying: “…when I come around your computer and I say drag one of your vertices of your quadrilateral, the sides have to stay congruent when you drag it. How are you gonna make something with four congruent sides – that’s what we gotta think about it.”

Mr. Jones walked in the room and gave students feedback. He encountered that students most often constructed parallelograms or rectangles. They adjusted the sides so that the shapes looked like a rhombus/square. After asking students to drag the vertices of their constructions, Mr. Jones constructed an argument about why their construction was not a rhombus/square. For example, he gave the following feedback to a student who constructed a parallelogram.
That’s a parallelogram. One thing about it, I know these sides are congruent, these two sides are congruent (points at the opposite sides), but are the four (sides) congruent to each other? Right? So, that’s a little bit an issue. If you drag it to the exactly right place, it might be. Uh, it’s not really gonna make a rhombus, though.

Three students were able to construct a rhombus or square. Mr. Jones invited a Sherpa student, Vince (pseudonym), to demonstrate his construction. Mr. Jones asked him to show all hidden in his construction. Then, Mr. Jones took the responsibility of connecting mathematics with the Sherpa student’s construction. Initially, he asked students to construct a square and rhombus. Students needed to ponder what properties they needed to take into consideration to accomplish the tasks. However, as the below vignette indicates, when the Sherpa student demonstrated his construction, Mr. Jones made a statement about the technological representation (circles with equal radius) that was related to a mathematical idea. From there, students observed that the Sherpa student used the *Circle* tool. Techniques Mr. Jones utilized were instrumental and mathematical. He said:

Mr. Jones: Vince show us the hidden things behind the square in the right hand side. Look at what he is doing. So, even though it’s talking about making a quadrilateral, what shape is he basing the quadrilateral off of?

Students: Circles.

Mr. Jones: Circles. I mean who would have thought that maybe you could have used a circle for this. Yeah, you can actually do this. What he is doing is he is intersecting two circles over here. Vince, do you remember how you made this guy over there so that the circles made that square there? You used two circles, were they special there?

Vince: I put the circles on the from like the radius…

Mr. Jones: Center of the other circle, right?

Vince: Uh-huh.

Mr. Jones: Yeah, so like their center is in common. We haven’t talked about much about circles. I know that the terminology there is, with the radius that kind of idea. Yeah, the radius is the same for both of those guys (circles). Yeah, I really liked that idea. And then, he hid a lot of things. But he had to do another thing. Not only did he get the segment across the bottom, what is it about these two lines that are going vertically? How did you construct those?

Vince: Because you have it like (pauses). Well, you construct it like (inaudible) perpendicular lines.

Mr. Jones: That’s kinda I am going for there. He made it so that there is a right angle down there the angle $D$. And, the right angle, angle $E$. And then, once you got that, then you can just go straight across the top and connect $B$ and $C$. And, you’ve got yourself a square, which is a type of rhombus.

For the rhombus construction task, they constructed an over-constrained rhombus whose opposite angles were fixed at 60 and 120°. Mr. Jones gave Vince step-by-step technology directions. Then, he used the Whiteboard and provided more information about constructions. He focused on mathematics and drew the line of reflection for illustrating how a rhombus could also be constructed by reflection of an isosceles triangle (Fig. 8).

In instrumental reinforcement, students were confronted with an instrumental difficulty. They were asked to use their mathematical knowledge to overcome this instrumental difficulty. In this instrumental integration, the task was created with a focus on
mathematics with the use of technology. The teacher took the responsibility of discussing the mathematical idea underlying the technological representation and provided alternative methods for constructions.

3.4. Instrumental symbiosis

In instrumental symbiosis, teachers aim at improving mathematical and instrumental knowledge with little instrumental difficulties. We identified instances in which teachers guide students to do constructions utilizing student-centered approaches. Teachers focused on mathematics with the use of technology. In his lesson, Mr. Sanders asked students to construct special quadrilaterals. They were guided to use the Perpendicular Line and Parallel Line tools, which allowed them to reason about the properties of special quadrilaterals. In the menu bar, these tools were not available to use in GSP unless a line/line segment and a point were highlighted. In the below excerpt, Mr. Sanders guided a student to construct a rectangle. He asked the student to highlight the line segment that was the top of the rectangle and identify where she wanted to construct a perpendicular line. The student employed a gesture to indicate where she wanted to construct a perpendicular line (Fig. 7a, b). Afterwards, she used the Perpendicular Line tool.

Mr. Sanders: This is the top of my rectangle. Tell me what do you know about this side. What do you want it to be?
Student: Perpendicular.
Mr. Sanders: OK. So, you want it to be perpendicular to this line (segment). What point do you want it pass through?
Student: (student employs a gesture as shown in Fig. 8a, b)
Mr. Sanders: There you go. Highlight that – where you want it perpendicular, too. And where do you want it to pass through? And now you’ve given [the information to] Sketchpad. All you need to do that.

There were several instances in which students made freehand constructions. Mr. Sanders explained why their constructions were not right and/or asked follow-up questions and students reconstructed the objects. Also, he asked students to recall mathematical definitions/properties, so students could identify required information to construct objects. For example, in the below excerpt, Mr. Sanders noticed the student drew a freehand trapezoid (Fig. 9a, b). Then, he asked the definition of a trapezoid and explained why the construction was not what he wanted. However, he provided step-by-step directions that were characterized in instrumental initiation after he noticed the student knew the definition of the trapezoid. The student was observed to make slow progressions on the construction tasks. Being satisfied with the student’s mathematical knowledge, Mr. Sanders might have given technology directions to help her keep pace with her classmates.

Mr. Sanders: Grab that red point there, and move (Fig. 10a). (The student drags the point as shown in Fig. 10b).
Mr. Sanders: I do see one problem, though. Define a trapezoid for me.
Student: A four-sided quadrilateral with exactly one [pair of opposite] parallel sides.
Mr. Sanders: Exactly – so which two are parallel in this (points at Fig. 10b)?
Student: Neither one.
Mr. Sanders: Right. You see that. When I moved it, it’s painfully obvious that we don’t have any parallel. We have a quadrilateral. I can move it so that it looks like a trapezoid, right? But it doesn’t pass the drag test. So, the thing I think you left out that we need to start with on this that is, we need to start out with some sides that we know that are parallel. So, do you know how to do that?
Student: Uh-huh.
Mr. Sanders: Alright, let’s do that. Give me a top and bottom that’s parallel…

In instrumental symbiosis, the relation between instrumental and mathematical knowledge was maximal. Teachers focused on
mathematics with the use of technology and guided students to use their mathematical knowledge to accomplish the given tasks. Instrumental abilities and mathematical knowledge were utilized to solve the tasks.

4. Discussion

In the current research study, we analyzed teachers’ organization of instrumental genesis utilizing student-created dynamic geometry sketches. The results indicated differences (Table 1). We observed teacher-dominated actions in instrumental initiation where there was little focus on mathematics. The two methods teachers used were: providing step-by-step technology directions and giving a responsibility to a Sherpa student. In this mode, teachers made interventions to students’ instrumentalization in order to eliminate technological confusions. Students’ lack of familiarity with technology may have motivated these teachers to give technology directions. In addition, teachers’ comfort level with technology may be related to their mode of integration. They provided a written set of directions students follow and they acted as technology resource. Assude (2007) and Erfjord (2011) reported a similar finding the fact that teachers gave a limited opportunity to have students use their instrumentation focusing on the use of artifact. As a result, students are less likely to connect mathematics using artifact (Assude, 2007). There was a minimal relationship between instrumental abilities and mathematical knowledge as the focus of the teacher was technology per se. Although we observed several Sherpa student configurations to do a construction or make a drawing, similar enactment to Technology-demonstration orchestration was identified (Drijvers et al., 2010). In other words, teachers dominated the Sherpa student configuration. Talmon and Yerushalmy’s (2004) research study indicated that students had difficulty understanding relationships between objects if they followed the written directions during a student-created construction.

We observed instances in which students utilized instrumental exploration and connected mathematics and technology. We identified two methods of instrumental exploration: performing specific actions and exploring the tools in the toolbar/menu bar. In the former method, the focus was on a technological representation or tool that followed a piece of mathematical knowledge to be tinkered with. In other words, when a tool in the GSP was introduced to students, they associated a technological representation with mathematical knowledge. In the latter method, the focus was on mathematics with the use of technology. Students were asked to recall a definition, property or simple fact to accomplish the tasks and mathematical knowledge was in use before a new tool was utilized. Overall, in instrumental exploration, instrumental abilities were more dominant because teachers’ primary focus seemed to demonstrate new tools in student-constructed sketches. Buteau and Muller (2014) indicate lower modes of instrumental integration as a precursor for upper mode tasks. However, using digital tools influenced by instrumental genesis is noted to be slow for some students (Buteau et al., 2016).

We found that when teachers utilized upper modes of instrumental integration, they focused on mathematics with the use of technology (Table 1). This may be due a construction requires using the tools with a focus on considering its mathematical properties (e.g., definitions). Using their mathematical knowledge, students created technological representations and established a link between instrumental abilities and mathematical knowledge. In instrumental reinforcement, adding an instrumental difficulty in tasks gave students an opportunity to link mathematical and instrumental knowledge in order to accomplish the task. In other words, students were not instructed what tool they could use to accomplish the construction tasks, which added an instrumental difficulty. They developed new mathematical knowledge to identify what tool satisfied the geometric constructions. In instrumental symbiosis, instrumental and mathematical knowledge was linked. Teachers gave students precursor activities and focused on mathematics with the use of technology, so students can apply their previous learning to new situations. They guided students to do constructions exploiting their mathematical knowledge using their instrumental abilities.

Similar to previous research studies indicated, we observed students made over-constrained and/or under-constrained constructions (Finzer & Bennett, 1995; Scher, 2005). When teachers utilized the upper modes of instrumental integration, they viewed these moments an opportunity for mathematical learning. Teachers assisted students in recalling a definition or constructing a mathematical argument and focused on mathematics with the use of technology. In other words, they focused on the mathematics with the use of technology in the upper modes of instrumental integration. Ruthven et al. (2008) characterized teachers’ such moves as capitalizing, and depicted as a successful integration of technology into mathematics curriculum. Construction tasks should be considered as an important activity for developing students’ knowledge of both mathematics and technology (Laborde, 1998). However, how the teacher utilizes instrumental genesis influences mathematical learning (Assude, 2007). If teachers allow students to feel free to discuss, compare, try their own methods, etc., then an artifact becomes an instrument. On the other hand, over-emphasizing the use of artifact places mathematical learning in a secondary stage where the use of artifact is on the primary focus.

Lower and upper-mode Sherpa-student configurations were observed frequently. This may be due to the fact that the whole class
shared a common goal of accomplishing a task. In the lower modes, teachers dominated student actions and the goal was learning the basics of the tools, completing a construction or making a drawing. In the upper modes, teachers gave students an opportunity to demonstrate their work in technology. When teachers were at the upper modes of instrumental integration that the technology became another tool from which they selected to achieve a particular mathematical goal. They did not use technology for its own sake, but rather for a focused and targeted purpose.

We observed important transitions between the modes of instrumental integration. Teachers purposefully created confusion to motivate the use of a particular tool. However, if teachers see these confusions as problems, they give technological directions to eliminate the confusions utilizing lower modes of instrumental integration. Also, teachers at the upper modes of instrumental integration sometimes focused on technology and informed students about the use of the artifact. This may be due to teachers’ being willing to help students whose instrumental genesis progress was slow keep pace with the class. Division of students as familiar and not familiar with the artifact as Assude et al. (2006) suggested may not be the correct division. Also, teachers may notice that students possess mathematical knowledge. Therefore, they may assist students in mastering technology and practice.

5. Implications and future directions

Characterizing teachers’ actions across different modes of instrumental integration was not a primary goal of the study. Therefore, we did not select individual teachers as our phenomenon of interest. Rather, we made collective arguments with a focus on teachers’ instrumental integration. However, it is also important to know how a teacher’s utilization of instrumental integration differs across the modes. For example, we observed that teachers who utilized an upper mode was also found utilizing a lower mode of instrumental integration for the same task. One may wonder how teachers use of different modes of instrumental integration differs from task to task (e.g., student-created and teacher-created sketches) and student to student (e.g., low achievers and high achievers). One of the important questions to consider is: What are some motives for teachers to make shifts in the focus of the task or instrumental integration (e.g., individual students’ mathematical understanding or background, time constraints)?

Additional research is needed to demonstrate if it is important to begin with knowledge of the artifact. For example, how does beginning with a focus on mathematics and properties, and then learning the artifact while students are working on mathematics influence their performance of reasoning about technological tasks? Another avenue for researchers may include how a teacher’s instrumental integration differs if he or she enacts a teacher-created sketches. Teacher-created geometry tasks may require using less instrumental abilities for students than in student-created sketches. Then, teachers may focus on mathematics with the use of technology or technology to notice mathematics more frequently.

In the upper modes of instrumental integration, we did not observe that teachers focused on technology to notice mathematics. Teachers most often enacted student-created sketches that required using mathematical knowledge to accomplish the task. Therefore, focus on mathematics with the use of technology characterized the upper modes. Teachers’ foci in the modes of instrumental integration are limited to the number of participants and their enactment of student-created sketches. For example, we have found on case of instrumental reinforcement. Working with more participants may demonstrate more diversity in characterizing teachers’ instrumental integration and focus.

Researchers have raised questions about different interfaces of DGS (Mackrell, 2011; Talmon & Yerushalmy, 2004). For example, students need to give an input to GSP, so the construction tools are available to use. On the other hand, in Cabri Geometry II Plus, students can use the construction tools without giving an input to the program. This freedom may help students explore new constructions without teachers’ assistance. Future research may demonstrate how teachers’ instrumental integration differs if they use technological tools with different interfaces.

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